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16. Abstract (Limit: 200 words)

The solar cycle is a magneto-fluid-dynamical process whose intensity varies cyclically in a time of about eleven years. Its arrhythmias reveal it to be a chaotic process that has intermissions every few hundred years. Our aim in this project is to capture the essential physical mechanisms underlying this behavior and to describe it in a mathematically simple model. We have studied the mathematical form such models may take and seen the causes of intermittency. We have isolated the probable seat of the solar cycle in the shear layer recently detected by helioseismology just below the convection zone. We call this layer the solar tachyline because of certain analogies to the oceanic thermocline. Using the methods of bifurcation theory to describe the nonlinear dynamics of this layer, we have uncovered a spatio-temporal behavior like that of the butterfly diagram characterizing the sunspot cycle. And, finally, we have uncovered in the turbulence of the tachycline, a promising mechanism for the formation of sunspots that is linked to the processes of vortex formation in geophysical fluid dynamics.

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Table of Contents

Report Documentation Page i

1. General 1

2. Chaos and Intermittency 1

3. Solar Convection 3

3.1. The Outer Convection Zone 3

3.2. The Solar Core 3

4. The Solar Tachycline 4

5. Convection and Rotation 6

6. Waves of Solar Activity 6

7. Subconvective Turbulence 9

7.1. Making Sunspots 9

7.2. Transport Processes in Stellar Interiors 9

References 10

1. General

The long-range aim of our work on this grant has been to produce a workable model of the main features of the solar cycle. We understand this process to be chaotic and intermittent in view of the erratic, yet cyclic, appearance of spots on the sun, peaking about every eleven years. Both of these properties are technically defined terms, but they can be characterized in terms of simple models. A chaotic system is a *sensitive system* in that small influences on it produce significant changes in subsequent behavior. In the simplest chaotic systems, we see a certain pattern repeated over and over again, but never exactly in the same way twice. Early solar observers seeing this behavior, without knowing about chaos as such, wisely assigned the term *cycle* to the process they observed.

However, some chaotic systems, are also intermittent. That is, they may interrupt their cyclic activity to switch to another activity or, more typically, to inactivity. The sun is of this kind and it is generally agreed that the solar cycle switched off for seventy odd years in the time of Newton. We have been trying to understand this behavior at several levels.

We have made simple models of chaos that produce bursts of oscillations interrupted by periods of inactivity. Having learned how to make such erratic, deterministic systems of equations, we have been attempting to find the essential ingredients that cause the intermittency. At the same time, we have been trying to understand, how such complexity arises in the sun. In some sense, at least implicitly, the sun has made such models from the churning of its turbulent outer layers. This has lead us to explore the fluid dynamical processes of the solar convection zone. Finally, we have been at pains to understand how the sun actually goes about making a sunspot, the basic element of the cycle itself. Our progress in each of these realms has been as diverse as the topics they imply, but what we have learned is taking shape as we move toward the next phase of putting these parts together to produce a reasonable scenario of the sun's inner workings.

The general background and some of the key developments abstracted here were presented in a series of lectures by Spiegel and Zahn on Stellar Fluid Dynamics in the 1990 Summer Study Program in Geophysical Fluid Dynamics (WHOI-91-03, Woods Hole Oceanographic Inst.).

2. Chaos and Intermittency

Many known models of intermittency need to have a parameter tuned rather carefully in order to obtain the intermittent behavior. We have been making more robust models of intermittency in which the phenomenon occurs for broad ranges of the parameters. We have concentrated on models that reflect the solar situation,

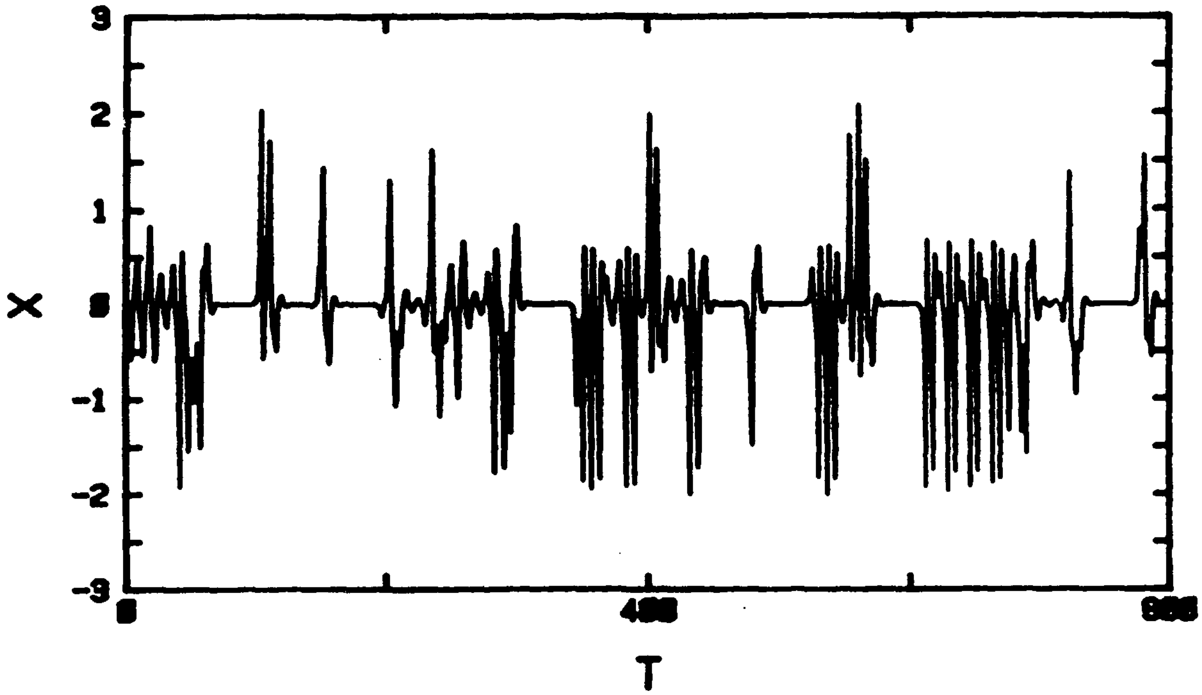

Fig. 1. Intermittent output from a deterministic system of five differential equations. The variable plotted is intermittent, but the other ones show continuous chaos.

in which the underlying convective turbulence continues always, while it produces cycles of activity only some of the time. Our recent work along these lines stems from a fifth-order system of differential equations devised some years ago [1], and whose output is shown in Fig. 1.

The original model gives the right kind of behavior, but it is complicated and it has been difficult to understand its workings. In collaboration with N. Platt (now) of the NRL and C. Tresser of IBM, we have isolated some essential ingredients for producing the form of intermittency seen in the figure. It suffices to couple a mildly unstable system, such as is commnonplace in bifurcation theory, to a conventional, simple chaotic system [2], just as the solar dynamo process is coupled to solar turbulent convection. The coupling in the model is of a form that replaces a parameter of the unstable system by a suitable variable from the chaotic system. This produces the desired kind of behavior quite simply. A description of this scheme is now being written, following a preliminary student report prepared by Platt for the proceedings of the 1990 GFD program cited above.

3. Solar Convection

The processes that drive most of the solar activity are the generation of nuclear energy in the core of the sun and the convective processes that arise as this energy tries to leave the solar core. These processes are under intense study by numerical specialists in various centers. Our aims have been to extract the essential features of this convection for the solar activity from the simulations, observations made at solar observatories, recent experiments such as those of Libchaber, and the kinds of elementary theory we describe in this section.

3.1. The Outer Convection Zone

The outer third of the sun is in vigorous convective motion and no adequate theory for such turbulent systems exists. Our immediate purposes are served by some understanding of the processes that go on just below the convection zone, for it is from here that we anticipate that the solar cycle is driven. Thus we confront the problems known in stellar structure theory as penetrative convection and overshooting. However, the stellar process has much in common with the laboratory and meteorological analogues [3].

As in those cases, there must be a region of mildly stable temperature gradient at the base of the outer solar convection zone. Zahn has been working on estimations of the extent of penetration into this zone and finds that it is of the order of the local pressure scale height, or 100,000 *km*. Zahn has written two papers giving his arguments on this matter and on other aspects of these issues:

— *Convective penetration into stellar radiation zones*, 1992. in *Challenges to Theories of the Structure of Moderate Mass Stars*, D.O. Gough and J. Toomre, eds. (Springer), 1992.

— *Convective penetration in stellar interiors*, *Astron. Astrophys.* (in press).

Zahn is presently working with a group at the Nice observatory to derive the solar acoustic frequencies that follow from this reasoning in order to compare them with the helioseismological observations.

3.2. The Solar Core

The solar core is the site of copious neutrino production in theory but, in practice, far fewer are seen than were expected [4]. Opinion about the cause of the discrepancy varies as some seek it in the solar models while others reexamine the theory of neutrinos. The most recent swing of this pendulum places the blame on

the neutrinos. If this resolution of the discrepancy is to teach us something about the actual properties of the neutrinos we need to have reliable models of the solar core. Partly for this reason, we have felt it worth reexamining the current belief that there is no convection in the solar core. For if there is convection in the solar core, it can generate magnetic fields that may bubble up and play a role in the solar cycle, and we need to be aware of this possibility.

If convection does occur in the solar core, it will not be driven in the conventional way. But core convection could be produced by temperature dependent nuclear heat sources. This topic was the object of a doctoral dissertation just completed by S. Ghosal.

In the course of the sun's evolution, a thin layer with an elevated abundance of He^3 is formed, whose nuclear burning is rather sensitive to temperature fluctuations. We found that the conditions in the solar core do favor a slow convection with a large horizontal scale, and this is documented in a paper by S. Ghosal and E.A. Spiegel:

— *On Thermonuclear Convection I. Instability Theory*, *Geophys. & Astrophys. Fluid Dyn.*, in press.

The nonlinear development of this instability gives rise to the possibility of waves travelling around the solar core and this aspect is described in paper II, whose manusucript is now being polished for publication. (Ghosal, incidentally, will go to the NASA Ames center at Stanford University where he will be employed to work on turbulence.)

4. The Solar Tachycline

Helioseismology has presented us with a picture of the internal rotation of the sun [5,6], so crucial to the dynamo process that powers the solar cycle. The rotation throughout the solar convection zone varies with latitude as it does on the surface, the angular velocity being independent of solar radius. The deep interior of the sun rotates rigidly, with a transition between the two behaviors taking place in a layer just beneath the convection zone that is too thin to have been resolved as yet. Our modeling of the solar cycle is based on the premise that this thin layer (called the solar tachycline, when its existence was first adumbrated nearly twenty years ago [7]) is the seat of solar activity. We have invested considerable effort in learning about its structure.

We have treated the rotation law of the subconvective sun in the manner of the oceanographer who oberves the prevailing winds and applies the resulting wind stress to the surface of the sea. From such considerations, he tries to deduce the nature of the ocean currents. In a like fashion, wc have adopted the observed motion of the convection zone and computed the resultant stress on the underlying layers.

The first effect of doing this is a transient wave of adjustment that propagates slowly into the solar interior and (we estimate) has now reached over half-way to the solar center by now. This process is like the one called spindown [7]. Just under the convection zone, the passing wave of rotational adjustment has left in its wake a region with strong differential rotation.

We estimate that the differential rotation under the convection zone has a large Reynolds number. But, the stable stratification in the vertical direction appears to be strong enough to inhibit vertical motions. Hence, we do not expect the stressed layer to spread in depth. On the other hand, the horizontal turbulence develops with little impediment. This layer of strongly anisotropic turbulence is the solar tachycline. It is thin because of the anisotropy of the turbulence, having a depth of a few tens of thousands of kilometers. The precise thickness of the tachycline cannot be predicted until we understand turbulence better. We hope that expected improvements in the resolution of helioseismology will soon permit verification or rejection of these results. In the meantime, these ideas have been presented at meetings and in an abstract of them prepared by Spiegel and Zahn:

— *The Turbulent Tachycline*, *Publ. Astr. Soc. Pac.*, in press.

A more extensive discussion is being prepared for publication.

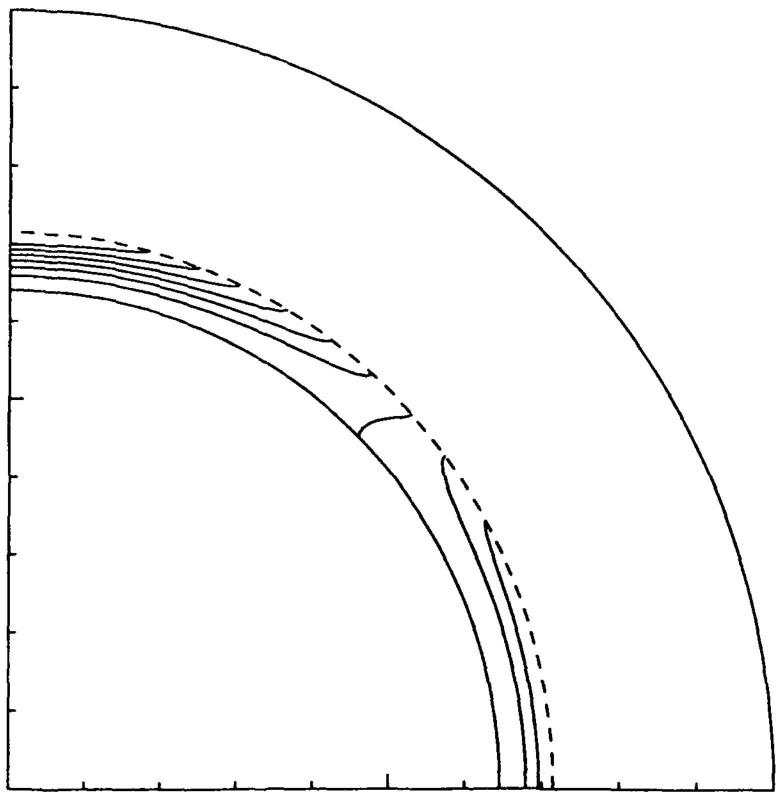

Fig. 2. Contours of the constant angular velocity in the solar tachycline.

5. Convection and Rotation

As a first approximation, we have been content to accept the observed large-scale motion in the convection zone as given in studying the tachycline. Nevertheless, we seek models of rotating turbulence that may capture the essence of the flows in the convection zone itself. In the simplest version, we can describe the turbulence in the convection zone as if it were a porous medium [8].

Zahn, with M. Rieutord (Observatoire de Toulouse), has revisited this idea, making the porosity anisotropic, in keeping with the strong descending plumes found in recent computer simulations of solar convection [9]. Their paper on this is:

— *Effects of turbulence on large scale motions: towards a model of coherent structures*, *The Sun and Cool Stars: Activity, Magnetism and Dynamos*, D. Moss, G. Rüdiger and I. Tuominen, eds. (Springer-Verlag) (1991).

We have also attempted to go beyond this simplified model and have looked into the coupling between shearing motions such as we have in the differential rotation and convection. Experiments in laboratory convection reveal that there is a spontaneous symmetry breaking of two-dimensional convection in which the convective cells deform so as to produce a large-scale shearing motion [10]. In turn, this shear, maintains the deformation in a rectified way that is self-sustaining for both components, the cells and the shear. We believe that a version of this process may be crucial for the driving of the solar differential rotation.

The onset of the observed excitation of shear by convection occurs for only highly unstable temperature gradients in the case of two dimensional motions. On the other hand, the gradient in the convection zone is nearly neutral once the turbulence has done its work. So we have examined the possibility of exciting shearing motions by convection and find that this is possible with weak gradients when the convection is three-dimensional. Our work on this is presented in a short paper written by Spiegel and Zahn with J. Massaguer of Barcelona:

— *Convection-Induced Shears for General Planforms*, submitted to *Physics of Fluids*.

6. Waves of Solar Activity

The solar cycle is believed to be a variation of magnetic activity, driven by rotating, turbulent convection. There are many ingredients in this process, and many versions of them, but there is some general agreement on how it all works. Differential rotation distends a seed magnetic field into a toroidal topology. Then, rising, twisting buoyant thermals turn this field into a poloidal structure, which in turn is made toroidal again by the shear.

For decades, it was supposed that this process takes place in the convection zone, and it is likely that there is some dynamo action throughout that zone. But it is hard to understand how the strong magnetic fields seen in sunpots can be formed in the convection zone itself, for they would be too rapidly ejected from the zone by rising motions whose buoyancy is enhanced when the field begins to strengthen. For this reason, we seek the origin of the solar cycle in the tachycline, where the stable density stratification can keep the field down until it is too strong to be confined, and then it rises to protrude in sunspots.

Our qualitative picture of the development of magnetic fields in the tachycline is that the plunging plumes from the convection zone penetrate into the tachycline, injecting a jumble of field lines. The conventional $\alpha - \omega$ dynamo then operates on this field. In this dynamo, the polodoidal and toroidal magnetic fields generate each other according to some simple linear equations [11]. These equations produce an instability in the form of growing oscillations, giving way to a Hopf bifurcation.

The dynamo theory mentioned provides a kinematic mechanism; it assumes certain motions and finds the magnetic fields that they generate. But these fields will feed back on the motions and the correct theory must be nonlinear. The complications involved in generating such a theory from first principles must be faced in time but, in a first look at the problem, we may use techniques from bifurcation theory to discover the form of the governing equations, once the linear theory has been given [12]. This technique has been used to derive nonlinear equations for the dynamo action of the tachycline in a paper written by Spiegel with M.R.E. Proctor (Cambridge University):

— *Waves of Solar Activity*, in the *The Sun and the Cool Stars, Activity, Magnetism and Dynamos*, D. Moss, G. Rüdiger and I. Tuominen, eds. (Springer-Verlag), (1991).

In fact, the equation found is a generalization of the so-called time-dependent complex Ginzburg-Landau equation, the generalization consisting of an allowance for the dependence of the parameters of the process on latitude. In the aforementioned paper, a solution of this nonlinear equation was developed by perturbation theory. In the limit of zero dissipation and no latitude dependence, the equation becomes the famous nonlinear Schrödinger equation, which admits solutions in the form of solitons. In such solitons there are three free parameters, the position, the velocity and the strength of the soliton. For the case of a single soliton, we used a version of singular perturbation theory [13] to derive equations of motion for these three paramters which are made to vary in the presence of dissipation and latitude variation. This was done for the relatively simple case of one soliton. But what is the meaning of this soliton in the solar problem?

The linear dynamo theory tells us that under given conditions, a band of hydromagnetic waves can become unstable. In the nonlinear theory, we consider a coupled packet of these waves and derive an equation for its envelope, which provides a

coarse-grained description of the instantaneous distribution of magnetic field. The occurence of a packet, in this axisymmetric model, shows how the activity is restricted in latitude and how this band of activity propagates as a solitary wave whose spacetime diagram is a reasonable representation of the butterfly diagram of the sun.

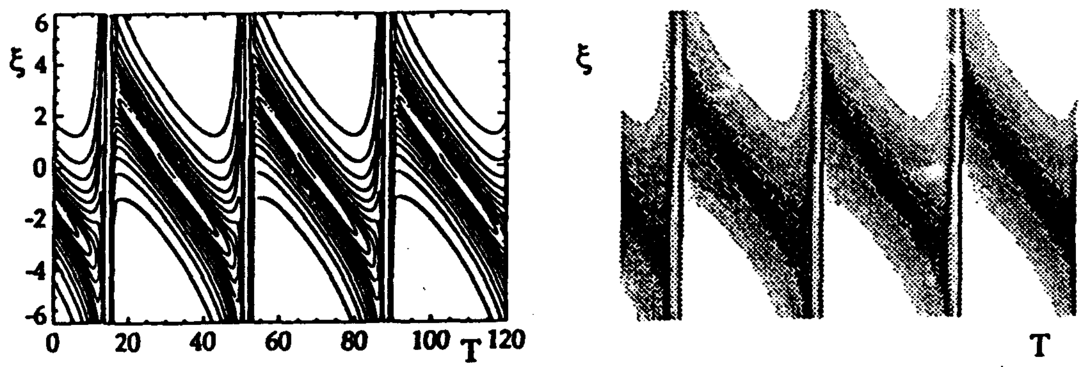

Fig. 3. Spacetime behavior of the solar solitary waves from the nonlinear dynamo theory. This model's the behavior in one hemisphere, with $\xi =$ latitude and $T =$ time.

The butterfly diagram shows the bands of solar activity as a function of latitude and time. It looks indeed like a row of butterflies. The results obtained in our solitary wave, while quite reminiscent of this pattern, is deficient in one notable respect: our butterflies are single-winged. This deficiency is a result of our having based our perturbation solutions on a single soliton rather than several. Each soliton corresponds to a wing of the butterfly, so we should have at least two. But when we put in more than one soliton at the outset, we have to deal with their interactions in deriving their equations of motion. Thus we need to undertand the dynamics of such coherent structures in interaction.

We have invested considerable effort in the study of dissipative nonlinear field theories in order to learn reliable ways to derive equations of motion for the interacting solitary waves that describe the global solar activity. Our results are developed in several papers, the most recent having been written with C. Elphick (Santiago), G.R. Ierley (MTU) and O. Regev (Technion):

— *Interacting Localized Structures with Galilean Invariance*, *Phys. Rev. A* **44** 1110-1122, (1991).

We have now used the methods developed for treating the coherent structures to study the dynamics of two wave packets in the two solar hemispheres. The result is a dynamical system describing the motion and intensity of the two "wings" of the butterfly. The dynamics described by this system is rich and clearly chaotic. We are optimistic that this is the first version of the model we seek for a chaotic solar cycle.

7. Subconvective Turbulence

7.1. Making Sunspots

The activity wave theory tells us when and where we may expect to see spots on average. Yet we do not know how a spot is actually produced. In the past year, we have begun to develop a mechanism of spot formation, on the basis of the dynamics of the tachycline. Being a thin, rotating layer, the tachycline is very like the thin layers studied in geophysics and planetary physics, and it will have much in common with these objects. Though there are differences as well, the basic fluid dynamics is very similar, and we may be guided to some extent by large scale simulations of rotating, turbulent fluid layers.

Rapidly rotating fluids behave essentially two-dimensionally, so that rapidly rotating turbulence closely resembles two-dimensional turbulence. The formation and development of vortices is perhaps the most striking feature of such free flows [14]. There are many further complications in these questions that we must omit here. For now, we adopt the current wisdom of theories of rotating turbulence [15] and assume that vortices will also form in the tachycline.

Given that the tachycline can be modeled as a thin layer of turbulent rotating fluid, we have adapted a working simulation of such a fluid that produces vortices and asked what it will do to a mild tordoidal field. The work has been done with S.P. Meacham (now at F.S.U.). We find that the vortex wraps up the magnetic field locally and expells it, as in previous calculations in magnetoconvection. We believe that this is the process that seeds the formation of the sunspot.

Since inhomogeneities in a fluid are rapidly smoothed out by pressure waves, and a region of strong field has high magnetic pressure, the local gas pressure is low where the field is strong. Hence so will the local density be low in the region of the expelled field. The expelled donut of field will therefore be subject to what Parker calls magnetic buoyancy and will float up out of the tachycline into the overlying convection zone. As it continues its rise, the vortex below will continue twisting up the large scale field and extruding it. Hence, a long, helical column of field will be driven up through the convection zone to emerge from the solar surface and form a spot. Our hope is to simulate this three-dimensional process, but it is costly and we are at present engaged trying to construct smplified models of it.

7.2. Transport Processes in Stellar Interiors

The radiative interior of a star is not wholly quiescent, even without the intercession of convection. Turbulence below the convection zone will have a strong influence on the processes we have been discussing. So it is fortunate that evidence for such motions is provided by anomalies in surface abundances of certain chemical species.

The mechanisms of matter and momentum transport by turbulence and waves has been the object of several reports at a sequence of conferences by Zahn:

— *Theory of transport processes*, in *Inside the Sun* G. Berthomieu and M. Cribier, eds. (Kluwer Acad. Publ.) 425, (1989).

— *Turbulent transport in stellar radiation zones: causes and effects*, *Rotation and Mixing in Stellar Interiors*, M.-J. Goupil and J.-P. Zahn, eds, (Springer), 141 (1990).

— *Turbulent shear flow and rotation*, *New Windows on the Universe*, M. Vasquez and F. Sanchez, eds. (Cambridge Univ. Press), 291 (1990).

We suspect that the turbulence in the region between the solar core and the outer convection zone is produced by shear instabilities due to differential rotation. These instabilities are typically due to so-called finite amplitude instabilities, and Zahn, together with B. Dubrulle (Observatoire de Toulouse), has examined these both analytically and by computer simulations to determine the instability threshold in:

— Non-linear instability of viscous plane Couette flow. I. Analytical approach to a necessary condition, *J. Fluid Mech.* (in press).

With B. Chaboyer (Yale University), Zahn has gone on to analyze the inhibition of the transport of chemicals by a large-scale circulation by a mechanism analogous to the turbulent shear dispersion discovered by G.I. Taylor in:

— *Effect of horizontal diffusion on the transport by meridional circulation*, *Astron. Astrophys.* (in press).

REFERENCES

[1] Spiegel, E.A. 1981.*A Class of Ordinary Differential Equations with Strange Attractors*, in *Nonlinear Dynamics*, R. H. G. Helleman, ed. *Ann. N.Y. Acad. Sci.*, **617**, 305-312.

[2] Moon, F.C. 1987. *Chaotic Vibrations*, J. Wiley and Sons, N.Y., Chapt. 3.

[3] Moore, D.W. 1967. *Summary-Introduction*, in *Aerodynamic Phenommena in Stellar Atmospheres*, Academic, LOndon) 405-414.

[4] Bahcall, J. 1989. *Neutrino Astrophysics*, (Cambridge Press).

[5] Brown, T.M., Christensen-Dalsgaard, J., Dziembowski, W.A., Goode, P., Gough, D.O. and Morrow, C.A. 1989. *Astrophys. J.* **343**, 526.

[6] Goode, P.R., Dziembowski, W.A., Korzennick, S.G. and Rhodes, E.J. 1991. *Astrophys. J.* **367**, 649.

[7] Spiegel, E.A. 1972.*A History of Solar Rotation*, in *Physics of the Solar System*,

S. I. Rasool, ed. Scientific and Technical Information Office, NASA, Washington, D.C. 1972, (NASA SP-300) Lib. Cong. Cat. Card No. 72-600107 p. 61. Theory of transport processes. *Inside the Sun (eds. G. Berthomieu and M. Cribier; Kluwer Acad. Publ.)*, 425 (1989)

[8] Bretherton, F.P. and Spiegel, E.A. 1968.*The Effect of the Convection Zone on Solar Spin-Down*, *Astrophys. J.* **153**, L77.

[9] Stein, R.F. amd Nordlund, Å, 1989. *Astrophys. J.* **342**, 195.

[10] Krishnamurti, R. and Howard, L.N. 1981. *Proc. Nat. Acad. Sci.* **78**, 1981.

[11] Krause, F. and Rädler, K..-H. 1980. *Mean-Field Magnetohydrodynamics and Dynamo Theory*, (Pergmon, Oxford) Chap. 15.

[12] Coullet, P.H. and Spiegel, E.A. 1988.*Evolution equations for extended systems*, in *Energy Stability and Convection*, G.P. Galdi & B. Straughan, eds., *Pitman Research Notes in Math.*, **168**, (Longman Sci. and Tech., N.Y.) 22.

[13] Bretherton, C.S. and Spiegel, E.A. 1983.*Intermittency Through Modulational Instability*, *Phys. Lett. A***96**, 152.

[14] McWilliams, J.C. 1984. *J. Fluid Mech.* **219** 361.

[15] Dowling, T.E. and Spiegel, E.A. 1990. *Stellar and Jovian Vortices*, in *Nonlinear Astrophysics*, R. Buchler and S. Gottesman, eds., (N. Y. Acad. Sci.), **617**, 190.